III. "The Measurement of the Power given by any Electric Current to any Circuit." By W. E. AYRTON, F.R.S., Professor of Applied Physics in the City and Guilds of London Institute, and W. E. SUMPNER, D.Sc. Received March 16, 1891.

I.

During the meeting of the Electrical Congress at Paris in 1881, one of us* devised a method of using an electrometer for measuring the power given to any circuit by any current. The accuracy of the method is wholly independent of the nature of the circuit, which may possess self-induction, mutual induction capacity, and an E.M.F., as well as of the nature of the current, which may be constant, intermittent, or alternating, according to any function of the time. This method is the only electrical one published up to the present date the accuracy of which is not based on assumptions, either as regards the nature of the current or as regards the entire absence of self- and mutual induction from a circuit some portion of which is necessarily of a solenoidal form, or as regards the nature of the circuit the power given to which we desire to measure.

In view then of the present wide use of alternating currents for industrial purposes, it might have been expected that this electrometer method of measuring the power given by any intermittent or alternating current to an inductive circuit would have been extensively employed. Unfortunately, however, as pointed out by one of us in conjunction with Professor Perry,† the use of this method is restricted by the fact that Sir W. Thomson's quadrant electrometers do not generally obey the mathematical law given for these instruments in text-books,‡ as it was supposed they did when this electrometer method of measuring power was first suggested. And hence the main result that has, up to the present time, followed from the publication of this method has been the stimulation of inventive minds to devise forms of electrometers in which the text-book law is strictly fulfilled.

In 1888, Mr. Blakesley published a very ingenious method for using three dynamometers to measure the power given by an alternating

* This method was simultaneously arrived at independently by Professor Fitzgerald.

† 'Journal of Soc. of Tel. Engs. and Elects.,' vol. 17, 1888.

‡ We may mention that an investigation on Quadrant Electrometers has been going on from time to time at the Central Institution for the last five years, and we had hoped to have communicated the complete report long before this to the Royal Society.

current to the primary coil of a transformer. His original proof, a geometrical one, was based on various hypotheses, amongst others, that the primary and secondary currents and the magnetic flux were sine functions of the time.

Recently, one of us, in conjunction with Mr. Taylor, has published* an analytical proof showing that Mr. Blakesley's three dynamometer method of measuring power gives equally true results, whatever functions the currents and magnetic flux be of the time. There still however, remains a serious objection to this method, viz., that it assumes the absence of magnetic leakage in the transformer, or in other words, that the number of lines of force embraced by one convolution of the primary coil at any moment is the same as the number of lines of force embraced by one convolution of the secondary. Further, the three dynamometer method cannot be used to measure the power given to a single circuit, as the coils of one of the dynamometers have necessarily to be put in different circuits.

The employment of an electromagnetic wattmeter for the measurement of electric power is well known, and investigators have considered the error that is introduced into wattmeter measurements made with alternating currents on account of the fine-wire circuit of the wattmeter possessing self-induction. This fine-wire circuit usually consists of a suspended coil in series with a so-called non-inductive stationary high resistance, and various devices have been adopted by different experimenters to make the effective self-induction of this fine-wire circuit nought. One of the simplest of these devices we venture to think is that proposed by one of us in conjunction with Mr. Mather, and which consists in winding the stationary so-called non-inductive resistance in such a way that the capacity of this doubly-wound coil practically neutralises the effect of the self-induction of the suspended coil.

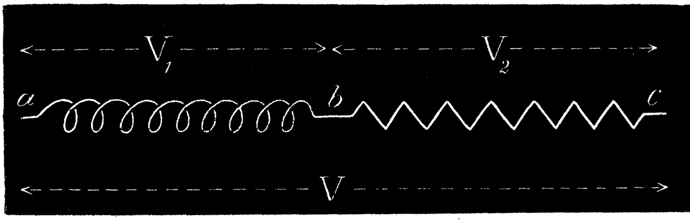
II.

Several months ago, however, while working at alternate current interference, we noticed that it was possible to employ an extremely simple method, based on the difference of phase of the P.D. and the current, for measuring the power by *any* current to *any* circuit. This method, which has since been in regular use in the laboratories of the Central Institution, is quite independent of any assumptions as to the nature of the current, or of the circuit, the power given to which it is desired to measure, and it has the further great advantage that the only measuring instrument required is the ordinary alternate-current voltmeter of commerce.

In series with the circuit ab (fig. 1), the power given to which we desire to measure, connect a non-inductive resistance bc of r ohms.

* Meeting of Physical Society, February 27, 1891.

Fig. 1.



Let V_1, V_2, and V be the readings of the voltmeter when applied between a and b, b and c, and a and c respectively; then, if W be the mean watts supplied to the circuit ab, we have in all cases, whatever the nature of the current, or of the circuit ab—

$$W = \frac{1}{2r}(V^2 - V_1^2 - V_2^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

For, let v_1, v_2, and v be the instantaneous values of the P.D. between a and b, b and c, and a and c at some moment t,

then $$v = v_1 + v_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

If α be the current in amperes flowing through the circuit at time t, then αv_1 equals the watts w given to ab at that time. But

$$\alpha = \frac{v_2}{r},$$

since the resistance bc is non-inductive;

$$\therefore w = \frac{v_1 v_2}{r}.$$

Then, squaring (2) we have—

$$v^2 = v_1 + 2v_1v_2 + v_2^2$$

$$\therefore w = \frac{1}{2r}(v^2 - v_1^2 - v_2^2).$$

Consequently $$\int_0^T w\,dt = \frac{1}{2r}\left(\int_0^T v^2 dt - \int_0^T v_1^2 dt - \int_0^T v_2^2 dt\right),$$

or $$W = \frac{1}{2r}(V^2 - V_1^2 - V_2^2),$$

the equation given above.

If the resistance of bc be not known, or if there be any fear that it may be changed by the passage of the current, then an ammeter (an

alternate current ammeter, of course, if alternate currents be employed) can be inserted in the circuit. Let A be the reading of this ammeter, and which represents the square root of the mean square of the current, then, for r in (1) we may substitute V_2/A, or

$$W = \frac{A}{2V_2}(V^2 - V_1^2 - V_2^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

When employing this last formula, the non-inductive resistance bc may be that offered by incandescent lamps, since there is no objection to the resistance varying with different mean strengths of the current employed.

This voltmeter method of measuring power was arrived at quite independently of the electrometer method referred to above, but an examination of the electrometer method shows that it is practically equivalent to simultaneous measurements of three P.Ds.

III.

The method which we have described for measuring the power given by *any* current to *any* circuit may be conveniently employed for measuring the power given to an alternating-current arc, or to an alternating-current arc-lamp. It is known that an alternating-current arc requires a greater current than a direct-current arc to produce the same light with similar carbons; for example, a 10-ampere direct-current lamp requires $12\frac{1}{2}$ amperes, or 25 per cent. larger current, when used with an alternating current. In a masterly paper on "The Theory of Alternating Currents," read before the Society of Telegraph Engineers, on November 13th, 1884, Dr. Hopkinson refers to a law given by Joubert, that the difference of potential between the carbons in an alternating arc is of approximately constant numerical value throughout the period, and that it reverses sign discontinuously at each reversal of the current. Using this law as his basis, he works out mathematically some very curious relationships between the variations of current and P.D. with time.

Three of our senior students, Messrs. Kolkhorst, Thornton, and Weekes, have been making a number of experiments on the power supplied to alternating-current arcs by using the method of measuring power described above. From these experiments it would appear that the quality of the carbon employed affects materially the difference in phase between the currents passing through the arc and the P.D. between the carbons. If the arc be quite steady and only give out the rhythmic hum that accompanies a good arc, such as can be obtained with cored carbons of proper quality, the arc appears to act practically as a simple resistance, and M. Joubert's law does not hold. But if the arc be maintained between uncored carbons of poor

quality, and be hissing, there is considerable difference in phase between the current and the P.D. between the terminals; further, the experiments show that current is very far from being a sine function of the time, although produced by a dynamo whose E.M.F. normally follows a harmonic law.

We do not purpose, in this communication, to enter at length into these experiments on alternate-current arcs, but a few examples of the experimental results that have been obtained will be interesting as illustrating the ready applicability of this new method of measuring power to such investigations.

In addition to the difference of phase of P.D. and current that may be produced in the arc itself, there is the electromagnet to be considered by which the distance between the carbons is usually regulated in arc lamps. This electromagnet will introduce lag between the P.D. at the terminals of the lamp and the current passing through the electromagnet and the arc in series; and hence, even although the arc be perfectly steady, we find, even in the case of a Brush lamp especially intended for alternate currents, that the true power supplied to the electromagnet and arc is 20 per cent. less than the product of the readings of the ammeter and the voltmeter attached to the lamp terminals, and which gives the square root of the mean product of the squares of the current and P.D.

If, however, the arc be between common carbons and be hissing, the difference, we find, is much greater. With cored carbons this Brush lamp requires a P.D. of about 35 volts to be maintained between its terminals, but if these cored carbons be replaced by common carbons and the arc be hissing, the P.D. between the terminals of the lamp at once rises to 45 or even 50 volts, although the current passing through the lamp and the amount of light given out remain practically as before. And then we find that the true power supplied to the lamp may be only one-half of the square root of the mean product of the squares of the current and P.D., so that the readings of the ammeter and voltmeter alone make the apparent power twice as great as the true power.

For the purpose of easily estimating the ratio of the true to the apparent power supplied, formula (3) may be thus written,

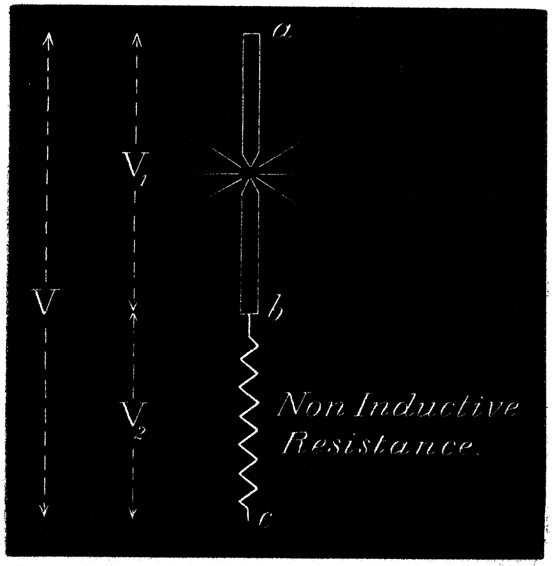
$$W = AV_1 \left\{ 1 - \frac{(V_1 + V_2 - V)(V_1 + V_2 + V)}{2V_1V_2} \right\} \quad \ldots\ldots\ldots\ldots (4),$$

from which we see that the expression in the brackets represents the ratio of the true to the apparent power supplied to the lamp or other circuit ab (fig. 1). Hence the percentage error made in assuming that the power supplied to any circuit was the product of the ammeter and voltmeter readings would be in all cases, whatever the nature of the current or of the circuit,

$$100\,\frac{(V_1+V_2-V)(V_1+V_2+V)}{2V_1V_2} \quad \ldots\ldots\ldots\ldots (5).$$

The following are samples of the results obtained with a hand-regulated lamp, there being no electromagnet at all in series with

FIG. 2.



the arc (fig. 2). The carbons were not cored and the arc was hissing. The frequency was maintained at 200 periods per second.

Table I.

Square root of mean square				Percentage error in estimating power formula (5).
Of P.D. in volts between			Of current in amperes.	
a and *b*. V_1.	*b* and *c*. V_2.	*a* and *c*. V.	A.	
55·0	60·0	108·0	12·3	24·0
45·4	75·4	107·3	11·8	45·8

For the purpose of obtaining an idea of ϕ, the angle of phase difference produced by the hissing arc, between the current and the

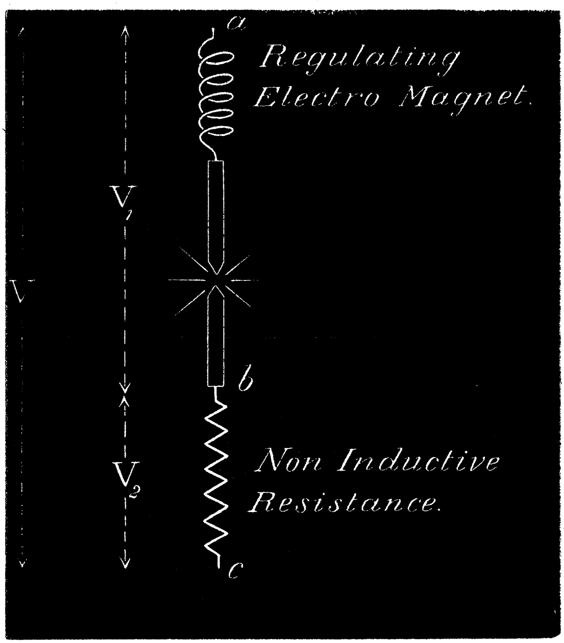
P.D., we may assume that the P.D. and current are sine functions of the time; then, as may be easily proved,

$$\cos\phi = \frac{V^2 - V_1^2 - V_2^2}{2V_1V_2} \quad \text{.................. (6)},$$

and the values of ϕ for the two tests given above come out as 40° 20′ and 57° 10′. It will, of course, be observed that this assumption of a harmonic law for the P.D. and current for the purpose of obtaining some idea of the value of ϕ in no way affects the generality of the method for the measurement of the power, since this is based on no such assumption.

The following are samples of the results obtained with a Brush alternate-current lamp regulated by an electromagnet (fig. 3), the

FIG. 3.



carbons not being cored, and the arc hissing. The frequency was maintained at 200 periods per second.

Table II.

Square root of the mean square				Percentage error in estimating power formula (5).	Lag between current and P.D.
Of P.D. in volts between			Of current in amperes.		
a and *b*. V_1.	*b* and *c*. V_2.	*a* and *c*. V.			ϕ.
64·8	58·0	108·4	13·0	44·0	56° 0′
59·8	64·2	107·4	12·0	50·5	60 20
55·0	67·3	107·4	10·6	47·0	58 30

The experiments already described tell us that a hissing arc may cause a considerable phase difference between the P.D. and the current, but they do not enable us to decide whether such an arc causes the current to lag behind the P.D., or to lead in front of it. To decide this point, that is, to decide whether a hissing arc acts like an inductive coil, or a condenser, a variety of experiments were made by putting induction or capacity in series with the arc. The following gives the result of one such experiment:—In series with a hand-regulated lamp (and, therefore, containing no electromagnet), was placed a condenser of 89 microfarads (fig. 4). Uncored carbons were used, and they were adjusted so that the arc was very short at first; the carbons were then not touched, and, as they burnt away, the arc grew longer and longer until it finally went out. The frequency was maintained at 200 periods per second.

FIG. 4.

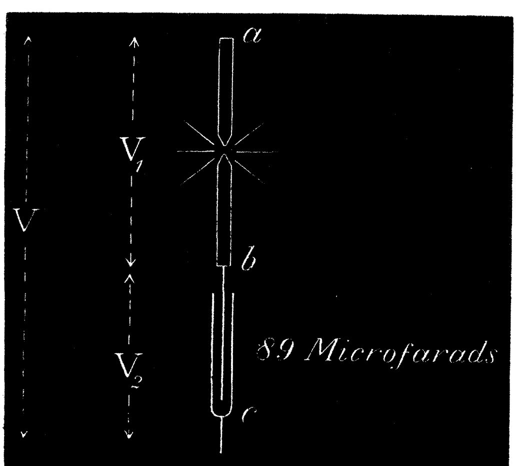


Table III.

E.M.F. of dynamo in volts.	Square root of mean square: Of P.D. in volts between a and b. V_1.	b and c. V_2.	a and c. V.	Of current in amperes. A.	Sum of $V_1 + V_2$.	Lag between current and P.D. ϕ.
59	35·4	89·0	72·3	12·0	124·4	129°
	38·0	92·0	73·3	12·5	130·0	133
	51·2	104·5	74·3	14·0	155·7	135
	69·2	86·5	67·5	13·4	155·7	131

Comparing V with the E.M.F. of the dynamo, we see that the arc and the condenser together acted as a condenser on the whole; but, comparing V with $V_1 + V_2$, we see that the arc acted as an induction and not as a capacity.

It having been conclusively proved that a hissing arc with uncored carbons acts as an induction, it was interesting to compare the impedance it produces with the impedance produced by the ordinary regulating electromagnet of the lamp. The arc itself seen in fig. 3 was, therefore, short-circuited, and the following measurements made, V_1 now being the square root of the mean square of the P.D. between the terminals of the regulating electromagnet, V_2 as before that between the terminals of the non-inductive resistance, and V that between a and c, the arc, as already explained, being short-circuited. The frequency was maintained at 200 periods per second.

Table IV.

V_1.	V_2.	V.	A.
35·4	69·2	82·0	11·3
35·4	65·6	80·0	11·4

We have, then, P.D. measurements giving the phase difference of current and P.D. with the arc alone (Table I and fig. 2); with the arc and regulating electromagnet (Table II and fig. 3); and with the electromagnet alone (Table IV). Defining impedance in the usual way as the ratio of the square root of mean square of P.D. to the square root of mean square of current, we find from the two sets of results given on Table I, that

the impedance of the arc alone equals .... $\begin{cases} 4{\cdot}47 \\ 3{\cdot}78 \end{cases}$;

from the three sets of results given in Table II, that

the impedance of the arc and electromagnet equals $\begin{cases} 4{\cdot}87 \\ 5{\cdot}15 \\ 5{\cdot}14 \end{cases}$;

and from the two sets of results given in Table IV, that

the impedance of the electromagnet alone equals.. $\begin{cases} 3{\cdot}14 \\ 3{\cdot}16 \end{cases}$.

In order to test whether the current follows a harmonic law, let us assume that it does, then find what result this assumption leads to, and, lastly, see whether the experiments confirm this result or not. Let, therefore, the instantaneous current at any moment be of the form

$$\frac{V_0}{\sqrt{(r^2+L^2p^2)}} \sin(pt-q),$$

where r is the effective resistance in each case, viz., the ratio of the *true* watts given to the circuit divided by the mean square of the current in amperes, and where p equals $2\pi n$, n being the number of periods per second. In each of the seven experiments referred to in Tables I, II, and IV, n was 200.

The seven values of r in ohms corresponding with the seven values of the impedance given above are for the

Arc alone .... $\begin{cases} 3{\cdot}42 \\ 2{\cdot}07 \end{cases}$.

Arc and electromagnet .... $\begin{cases} 2{\cdot}65 \\ 2{\cdot}66 \\ 2{\cdot}71 \end{cases}$.

Electromagnet alone .... $\begin{cases} 0{\cdot}44 \\ 0{\cdot}54 \end{cases}$.

And, since the impedance equals $\sqrt{(r^2+L^2p^2)}$, if the harmonic law be true, the corresponding values of Lp are, for

Arc alone .... $\begin{cases} 2{\cdot}88 \\ 3{\cdot}16 \end{cases}$.

Arc and electromagnet .... $\begin{cases} 4{\cdot}08 \\ 4{\cdot}41 \\ 4{\cdot}37 \end{cases}$.

Electromagnet alone .... $\begin{cases} 3{\cdot}12 \\ 3{\cdot}11 \end{cases}$.

But if the harmonic law hold for the current, the sum of Lp for the arc alone, plus the Lp for the electromagnet alone, must equal the Lp for the arc and electromagnet, since p has the same value in each case. Now it is obvious that the condition is far from being fulfilled with the numbers just given. Hence the current does not follow a harmonic law.

It is interesting to notice that the Lp for the hissing arc alone is actually greater than the Lp for the regulating electromagnet.

The values given above for r, being obtained by dividing the *true* watts by the mean square of the current in amperes, are the effective resistances in ohms—whether the current follows a harmonic law or not. Hence, by comparing the value of r for the regulating electromagnet alone with its resistance in ohms, measured with a steady current, we have a true measure of the waste of energy in the iron core of the electromagnet due to hysteresis and Foucault currents. Now the resistance of this electromagnet for a steady current is only 0·065 ohm; hence 90 per cent. of the energy given to the regulating electromagnet of this Brush lamp is wasted in heating its iron core when the frequency is 200. Here again we have a further illustration of the importance of being able to measure, by means of the simple method we have described, the power given by any current to any circuit.

Added March 31, 1891.

IV.

The Best Value to give to the Non-Inductive Resistance.

In cases where great accuracy is required in the measurement of the power given to a circuit, it is important to consider what value should be given to the non-inductive resistance (fig. 1), in order to reduce to a minimum any error that may arise from possible inaccuracies made in the three readings of the voltmeter, or on the graduation of its scale.

Since
$$W = \frac{1}{2r}(V^2 - V_1^2 - V_2^2),$$
$$dW = \frac{1}{r}(VdV - V_1dV_1 - V_2dV_2),$$

where dV, dV_1, dV_2 are the errors made in the estimation of the three P.Ds.

Let
$$dV = \pm eV$$
$$dV_1 = \pm eV_1$$
$$dV_2 = \pm eV_2$$

where e is a small fraction, *i.e.*, let the errors be each the same small fraction of the correct value, then the probable value of $(dW)^2$ is

$$\frac{1}{r^2}\left(V^2e^2V^2+V_1^2e^2V_1^2+V_2^2e^2V_2^2\right),$$

so that
$$\left(\frac{dW}{W}\right)^2 = 4e^2\frac{V^4+V_1^4+V_2^4}{(V^2-V_1^2-V_2)^2} \quad\ldots\ldots\ldots\ldots (7).$$

Let the non-inductive resistance have such a value that

$$V_2 = xV_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8),$$

V_2 being already defined, the square root of the mean square of the P.D. between its terminals, and V_1 the square root of the mean square of the P.D. between the terminals of the circuit the power given to which we desire to measure. Then we wish to find the value of x that will make dW/W a minimum.

Let ϕ be the angle of lag between the current in the circuit ac and the P.D. at the terminals of ab (fig. 1), then ϕ is the angle of lag between the P.D. at the terminals of ab and the P.D. at the terminals of bc. Hence, since

$$v = v_1+v_2,$$

v, v_1, and v_2 being the instantaneous values of the P.Ds.,

$$V^2 = V_1^2+V_2^2+2V_1V_2\cos\phi \ldots\ldots\ldots\ldots\ldots (9).$$

Eliminating V, V_1, and V_2 by means of equations (7), (8), and (9), we have

$$\left(\frac{dW}{W}\right)^2 = 4e^2\frac{(1+x^2+2x\cos\phi)^2+1+x^4}{4x^2\cos^2\phi}.$$

Now $\cos\phi$ depends on the circuit, the power given to which we desire to measure, and is independent of x. Hence differentiating with respect to x and equating to nought in the usual manner, we find that x equal to unity makes $\dfrac{dW}{W}$ a minimum.

Hence, inaccuracies in the three readings of the voltmeter, or in the graduation of its scale, produce the least effect in this method of measuring power when the P.D. between the terminals of the non-inductive resistance is equal to the P.D. at the terminals of the circuit under test.

The next point to consider is, what is the percentage error made in measuring the power by this method compared with the percentage error made in reading one of the P.Ds.

Let x equal unity, then

$$\frac{dW}{W} = 2e\,\frac{\sqrt{2+4(1+\cos\phi)^2}}{2\cos\phi},$$

or

$$\frac{dW}{We} = \frac{\sqrt{2+4(1+\cos\phi)^2}}{\cos\phi}.$$

Now dW/We is the ratio of the percentage error made in measuring the power to the percentage error made in measuring one of the P.Ds. and the right-hand side of the last equation we find equals from 4 to 5 for the values of the lag angle ϕ that occur in ordinary practice. If then there were a positive or a negative error of 1 per cent. in each of the measurements of V, V_1, and V_2, there would be a probable error of from 4 to 5 per cent. in the measurement of the power. The probable percentage error in the measurement of the power being from 4 to 5 times the error in the measurement of each of the P.Ds. arises partly from the fact that the expression for W, being

$$\frac{1}{2r}\,(V^2 - V_1^2 - V_2^2),$$

depends directly on the difference in the mean squares of the P.Ds., and not on the difference of the square roots of the mean squares. And as all instruments that are graduated for measuring the square root of the mean square of an alternating P.D. such as a hot-wire voltmeter, an electrostatic voltmeter, &c., really measure the mean square and not the square root of the mean square directly, it would be better, if such an instrument were to be employed for the method of measuring power described in this paper, that it should be graduated in mean squares of P.Ds. and not in the square roots of the mean squares. In that case a similar line of reasoning to that employed above shows that the probable percentage error in the measurement of power by the method would be from 2 to 2·5 times the error in the measurement of each of the P.Ds.

It is, of course, clear that these errors to which we have been referring are not errors in any way essential to the method proposed for measuring power, since by the employment of an accurately graduated voltmeter, by exercising care in taking the readings, and if necessary, by repeating the measurements two or three times and taking the means of the observations, the power can be measured to any degree of accuracy desired.

V.

Approximate Calculation of the Power from the Three Readings of the Voltmeter.

The calculation of the power from formula (1) is easy, especially when the voltmeter is graduated to read the mean squares of the P.Ds. and not the square roots of the mean squares. If, however, as is usually the case, the scale is graduated in square roots, then even the trouble of taking the squares may be saved, when V_1+V does not differ much from V, by using the following method:—

Let the inductive resistance be arranged so that V_1 is nearly equal to V_2, and let

$$V_1+V_2-V = yV_1;$$

then, since

$$\cos\phi = \frac{V^2-V_1^2-V_2^2}{2V_1V_2},$$

we have by making V_1 equal to V_2 and eliminating V, V_1, and V_2 from the last two equations,

$$1-\cos\phi = 2y\left(1-\frac{y}{4}\right).$$

Now the power that would be given to *ab* (fig. 1) if there were no lag, or the apparent power, as it may be called, would be

$$\frac{V_1V_2}{r},$$

whereas the power that is actually given to *ab* is

$$\frac{V_1V_2}{r}\cos\phi.$$

Hence,

$$\frac{\text{the apparent power}-\text{the true power}}{\text{the apparent power}} = 1-\cos\phi$$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad = 2y\left(1-\frac{y}{4}\right)$$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad = 2y \text{ approximately}$$

if the lag be not very large.

For example, suppose V_1 or V_2 were 50 volts, and V were 98 volts, then y, or

$$\frac{V_1+V_2-V}{V_1},$$

would be 4 per cent. Hence the true power would be 8 per cent. less than the apparent power. Or, in other words, to find the true power given to ab (fig. 1), we should merely have to diminish V_1V_2/r by 8 per cent. and the answer would be obtained.

If r were unknown, and A the square root of the mean square of the current were measured instead, then to obtain the true power for the values of V_1, V_2, and V given above, we should diminish V_1A, the apparent power, by 8 per cent.

We will finally consider what is the percentage error made in estimating the power by the method last described, compared with the percentage error made in taking the value of V_1+V_2-V.

Let us assume that, on account of errors in the readings of V_1, of V_2 and of V, or on account of inaccuracies in the graduation of the voltmeter, the value of V_1+V_2-V is taken as half a volt greater than its true value, that is, that this expression is erroneously increased by 1 per cent. of V_1 if we assume V_1 to be 50 volts as above. Then y will be also increased by 1 per cent., and since the true power is obtained by subtracting from the apparent power $2y$ times the apparent power, it follows that the power measured in this way will be estimated as 2 per cent. too low if the combined error made in measuring V_1+V_2-V be *plus* 1 per cent. of V_1.

VI.

Measuring the Power given out by an Alternate-Current Dynamo.

In consequence of the trouble usually experienced in correctly measuring the power given to an inductive circuit, it is usual when measuring the power given out by an alternate-current dynamo to use for the outside circuits various resistances, all of which are as far as practicable non-inductive. But as the construction of adjustable non-inductive high resistances that will take large currents is a troublesome matter, we suggest the following as a convenient method of overcoming the necessity of employing such a non-inductive circuit :—

Let the circuit external to the dynamo be ac (fig. 1), only a portion of which is non-inductive; then, if V_1, V_2, and V have the values already given them, it is easy to show that the power given to both the inductive and non-inductive portion of, ac that is, to the whole circuit external to the dynamo, is

$$\frac{1}{2r}\,(V^2-V_1^2+V_2^2).$$

And we anticipate that, if only a small portion bc of the circuit be strictly non-inductive, this voltmeter method of measuring the power given out by an alternate-current dynamo will give more accurate

results than can be often obtained by assuming that a so-called non-inductive circuit is really non-inductive, and, therefore, that the apparent power is the true power.

IV. "On Galvano-Hysteresis. (Preliminary Notice.)" By SILVANUS P. THOMPSON, D.Sc., B.A., Professor of Physics in the City and Guilds Technical College, Finsbury. Communicated by Professor G. CAREY FOSTER, B.A., B.Sc., F.R.S. Received March 16, 1891.

1. If a sufficiently strong electric current is passed through a coil of insulated soft iron wire for a short time, and the wire then disconnected, and if, after the lapse of any length of time, the wire is placed in the circuit of a galvanometer, and is then subjected to longitudinal magnetisation or to a succession of alternately directed longitudinal magnetisations, it is found to discharge an electric current through the galvanometer.

2. The direction of the current discharged from the iron wire is found to be the same as that of the current which was originally passed through it.

3. The direction of the discharge current is opposite to that in which the discharge current would flow if the wire acted as a condenser.

4. A wire which has once produced such a discharge current will not produce a second unless again traversed by a charging current.

5. A wire which has not been subjected to any preliminary process of charging, that is to say, one which since being annealed has not been traversed by an electric current, does not sensibly show any such phenomena, either when subjected to longitudinal magnetisation or to a succession of alternate magnetisations.

6. The sense of the discharge current is quite independent of the direction of the longitudinal magnetisation used in producing the disturbance which effects the discharge.

7. The time-integral of the discharge current is independent of the duration of the charging current, provided this is not too suddenly turned off. It increases with the strength of the charging current up to a certain limit, being proportional to it through a certain range of values, but is not proportional to it for currents below or above certain limits of strength. These limits vary with the gauge of the wire, but are independent of its length. For a charging current of given strength the discharge current from a given wire is greatest if the charging current is gradually reduced to zero and not abruptly broken by a spark.

8. The time-integral of the discharge current is practically inde-

FIG. 1.

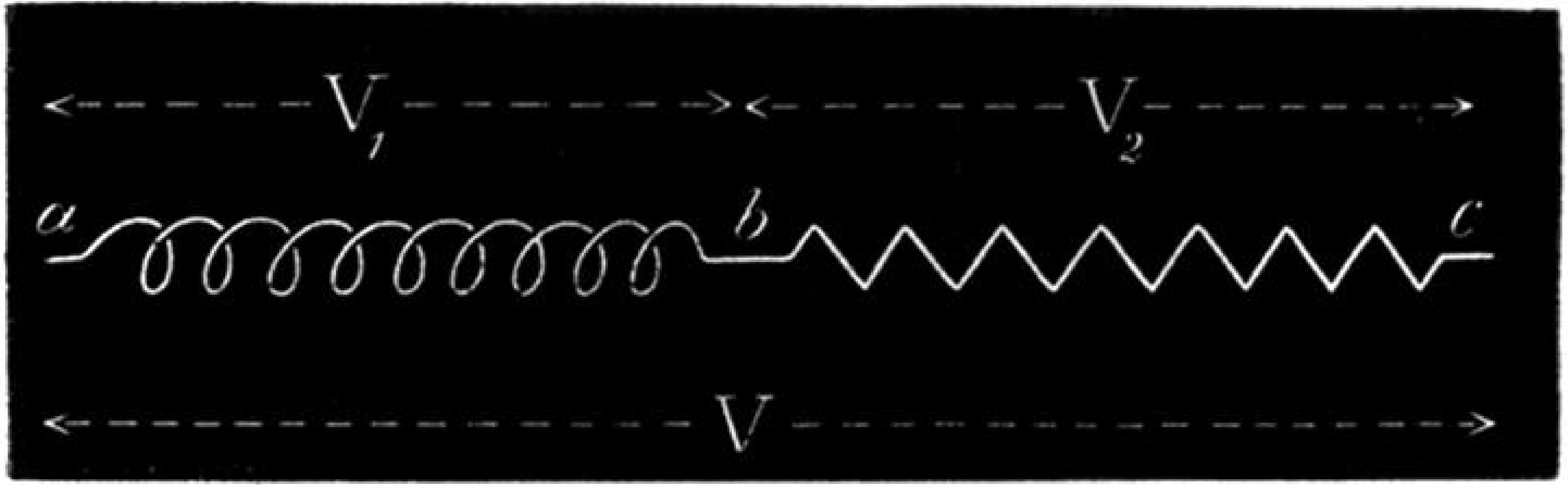


FIG. 2.

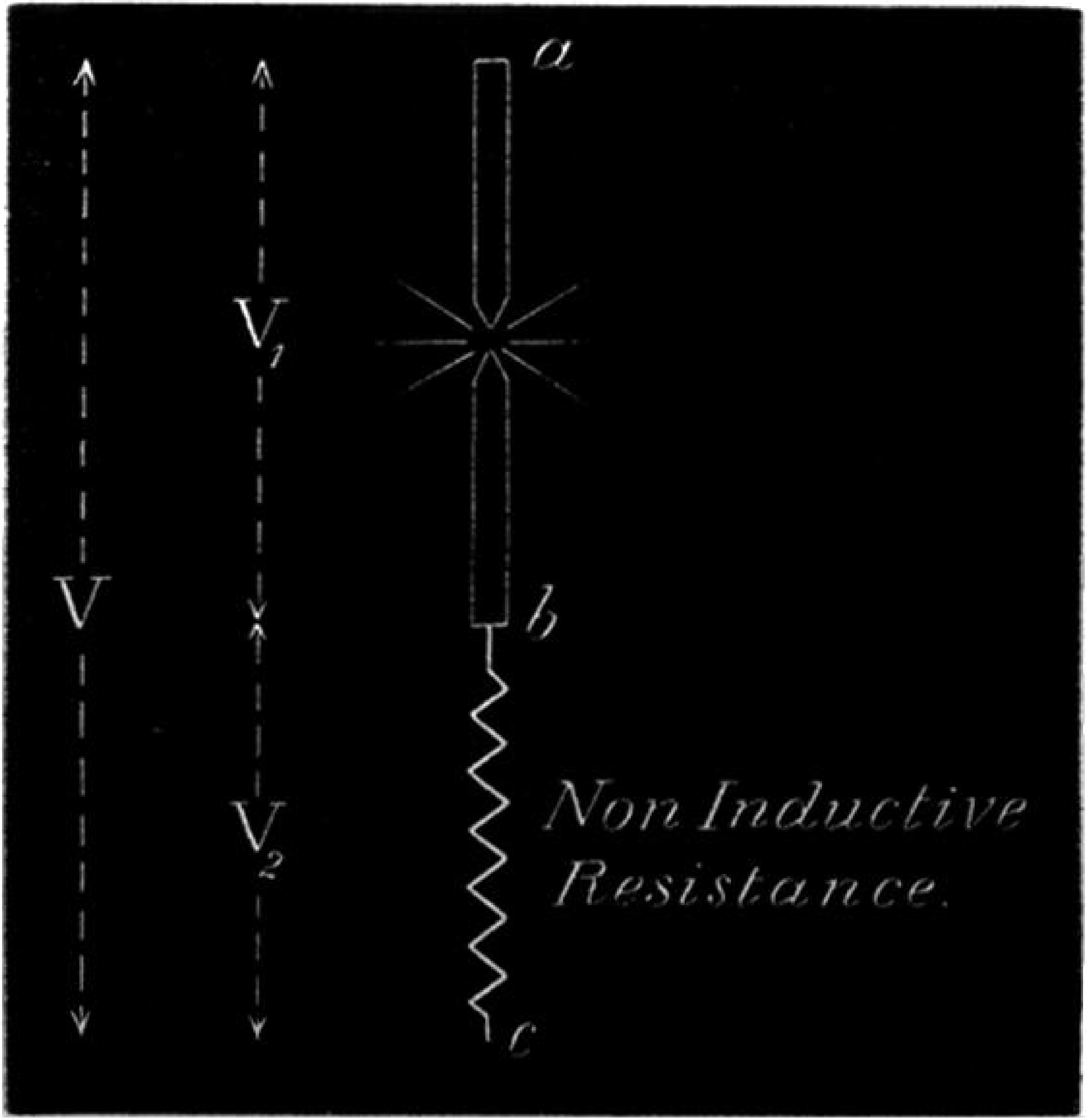


FIG. 3.

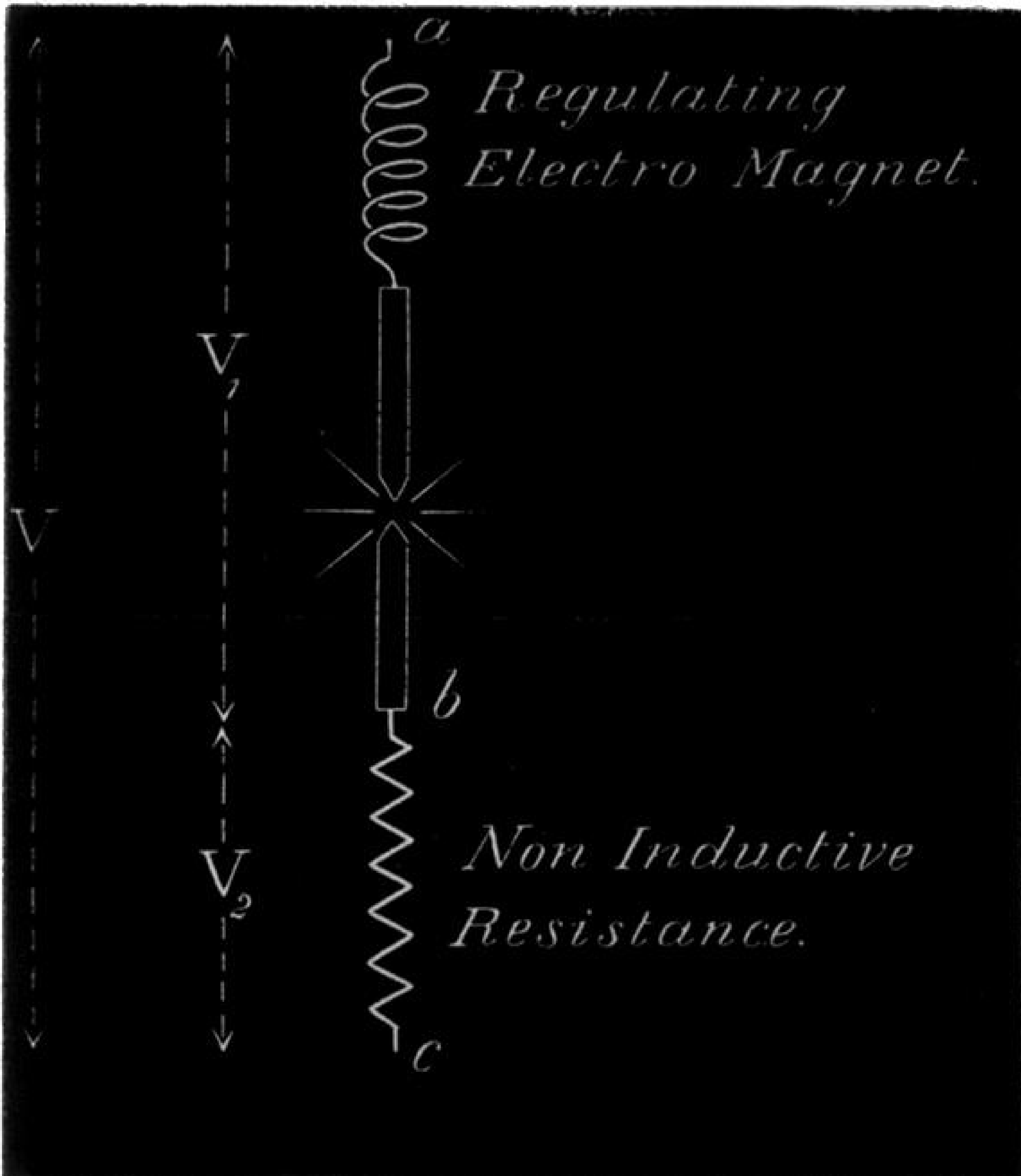

a
Regulating
Electro Magnet.
V_1
V
b
Non Inductive
Resistance.
V_2
c

FIG. 4.